Beer–Lambert Absorption Enhancement in Structural and Material Slow-Light

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Dipartimento
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dell'Informazione

Photonics and
Electromagnetics Group

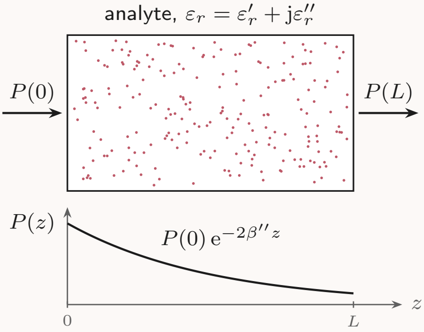
Beer–Lambert absorption and absorption sensing

- Transmitted power through an absorbing medium:

$$P(z) = P(0)\,\mathrm{e}^{-\alpha z}, \qquad \alpha = 2\beta'',$$

  for propagation constant $\beta = \beta' + \mathrm{j}\beta''$.
- Bouguer (1729), Lambert (1760): attenuation is exponential in path lenght. Beer (1852): α is linear in concentration [A. Beer, Ann. Phys. **162**, 78–88 (1852)].
- **Analyte**: absorbing gas, liquid, or dilute component,

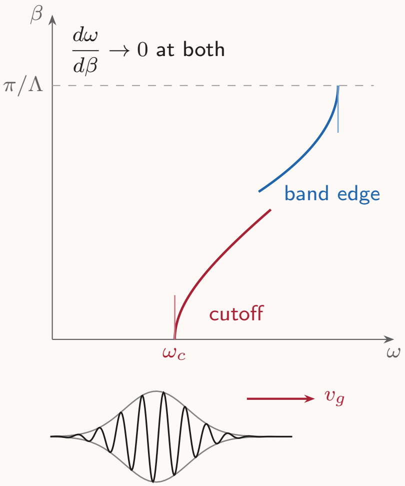
$$\varepsilon_r = \varepsilon_r' + \mathrm{j}\,\varepsilon_r'', \qquad \varepsilon_r'' > 0 \text{ represents loss.}$$



- For a dispersion relation $F(\omega, \beta) = 0$, define group velocity and index:

$$n_g = c\,\mathrm{Re}\left[\frac{d\beta}{d\omega}\right], \qquad v_g = \frac{c}{n_g}.$$

- **Slow light**: $n_g \gg 1$ (flat $\omega(\beta)$), from
  - material dispersion: atomic lines, EIT, Raman, Brillouin;
  - structural (waveguide design): band edges, resonators, cutoff.
- **Operational meaning** [J. Peatross et al., Phys. Rev. Lett. 84, 2370–2373 (2000)]: with real ω and complex $\beta(\omega)$, the Poynting-flux centroid delay over Δz is exactly $\langle \mathrm{Re}[d\beta/d\omega] \rangle\, \Delta z$, averaged over the output spectrum, plus a reshaping term that vanishes with the bandwidth.



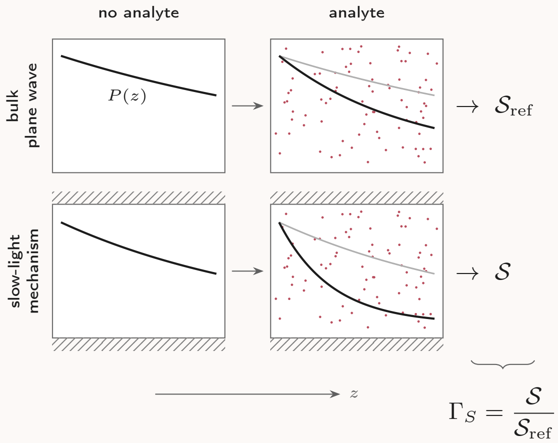
Does slow light necessarily enhance absorption?

- [S. Chin et al., Adv. Opt. Sci. Congress (2009)]
  [L. Thévenaz et al., Adv. Slow Fast Light V (2012)]
  [I. Dicaire et al., Opt. Lett. **37**, 4934–4936 (2012)]: **measured**: no enhancement in a Brillouin-slowed solid-core fibre with acetylene in its holes; enhancement in a structural photonic-crystal guide.
- [J. Grgić et al., Opt. Express **18**, 14270–14279 (2010)]: **computed**: photonic-crystal-fibre enhancement does not exactly follow n_g.
- [H. G. Winful, Phys. Rep. **436**, 1–69 (2006)]: for fixed-length evanescent systems, group delay is a dwell time, not a transit time.
- [E. O. Schulz-DuBois, Proc. IEEE **57**, 1748–1757 (1969)]: **energetic interpretation** propagating wave: $2\beta'' = \gamma_D/v_E$; v_E: energy velocity, γ_D: loss per stored energy.

Is n_g the index that measures absorption enhancement?

- **Loss sensitivity** at fixed ω, geometry, ε_r', ($k_0 = \omega/c$):
$$\mathcal{S} = \frac{1}{k_0}\,\frac{\partial \beta''}{\partial \varepsilon_r''}\,.$$
- **Reference** $\mathcal{S}_{\text{ref}}$: plane wave without slow light (structural: **analyte alone**; material: **gain line off**).
- **Enhancement factor**: $\Gamma_S = \mathcal{S}/\mathcal{S}_{\text{ref}}$.

Remark : One metric for **any linear slow-light mechanism** (any enhancement mechanism, more in general) with complex β.



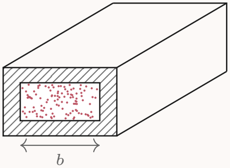
One dispersion relation, two physical cases

- $n = n' + \mathrm{j}n''$: uniform, non-dispersive filling index; $\varepsilon_r = n^2$.
- $k_\perp$: TE/TM transverse wavenumber; $n_\perp = k_\perp / k_0$. For PEC TE_{01}, $k_\perp = \pi / b$.

For $\mathbf{E} = \boldsymbol{\psi}(x,y)\,\mathrm{e}^{\mathrm{j}(\beta z - \omega t)}$ and $\nabla_\perp^2 \boldsymbol{\psi} = -k_\perp^2 \boldsymbol{\psi}$,

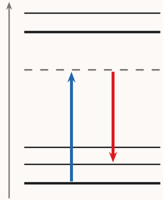
$$\beta^2 = k_0^2\, n^2 - k_\perp^2.$$

$k_\perp \neq 0$: structural



PEC walls fix $k_\perp$; analyte-filled guide.

$k_\perp = 0$: material



Plane wave; dispersion entirely in $n(\omega)$.

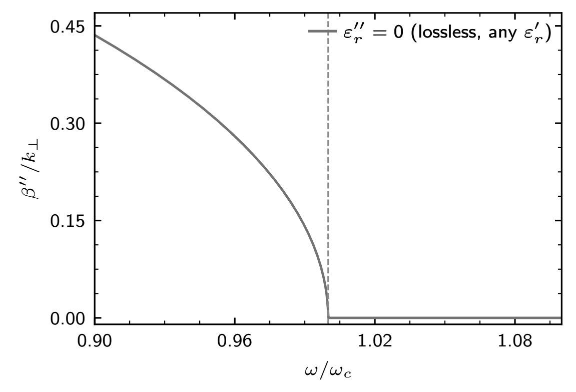
Analyte-filled PEC guide, TE mode:

$$n_{\text{eff}} = \frac{\beta}{k_0} = \sqrt{\varepsilon_r - n_\perp^2}.$$

Cutoff. Lossless filling: $\beta' = \beta'' = 0$ at $k_0^2 \varepsilon_r' = k_\perp^2$,

$$\omega_c = \frac{c\,k_\perp}{\sqrt{\varepsilon_r'}} \qquad \left(\text{TE}_{01}:\ \omega_c = \frac{\pi c}{b\sqrt{\varepsilon_r'}}\right).$$

$\omega > \omega_c$: propagating; $\omega < \omega_c$: evanescent, $\beta'' > 0$ even without loss. With loss, β remains complex through cutoff.



Structural slow light: a PEC waveguide with lossy dielectric near cutoff

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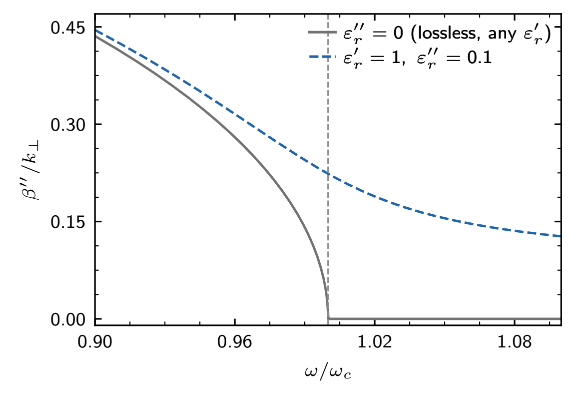

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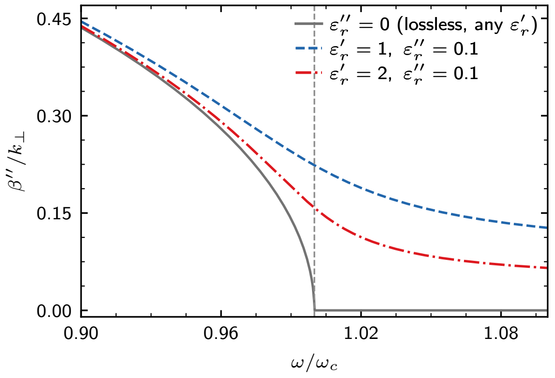
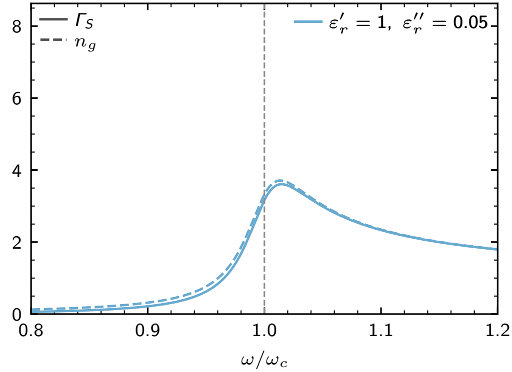


Non-dispersive filling ($dn_\perp/d\omega = -n_\perp/\omega$):

$$n_g^{\mathrm{wg}} = n'_{\mathrm{eff}} + \omega\frac{dn'_{\mathrm{eff}}}{d\omega} = n'_{\mathrm{eff}}\left(1 + \frac{n_\perp^2}{|n_{\mathrm{eff}}|^2}\right),$$

$\mathcal{S}^{\mathrm{wg}} = n'_{\mathrm{eff}}/(2|n_{\mathrm{eff}}|^2)$, $\mathcal{S}_{\mathrm{ref}} = n'_{\mathrm{ref}}/(2|n_{\mathrm{ref}}|^2)$ (plane wave, same ε_r).

	$\omega \ll \omega_c$	$\omega \gg \omega_c$
	evanescent limit	**plane-wave limit**
n_{eff}	$\to \mathrm{j}n_\perp$	$\to n$
$\beta''/k_\perp$	$\to 1$	$\to k_0 n''/k_\perp$
n_g	$\to 0$	$\to n'$
Γ_S	$\to 0$	$\to 1$



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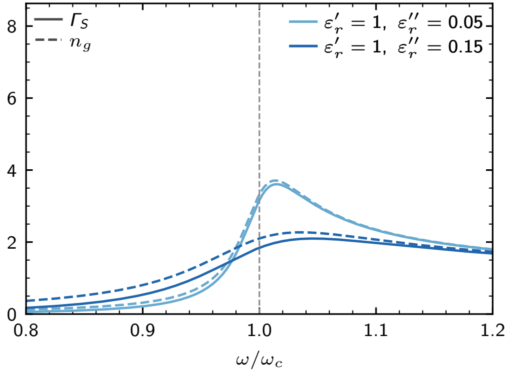

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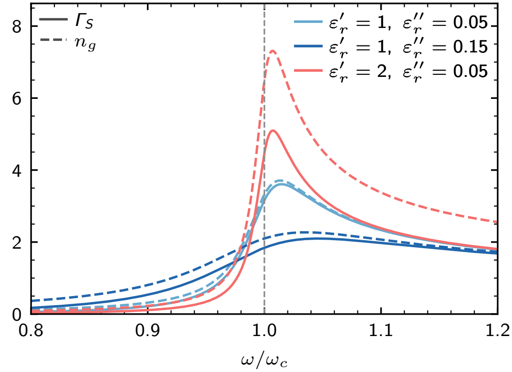

Loss sensitivity rises near cutoff, but is not equal to n_g

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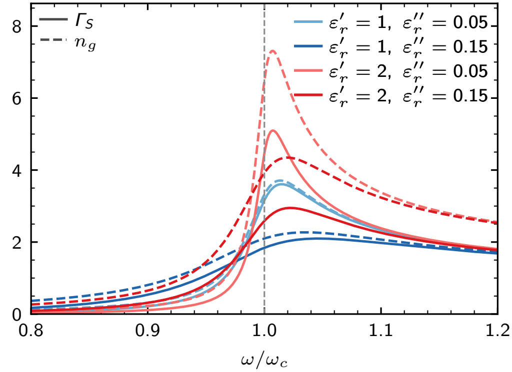
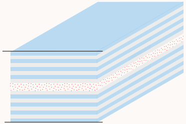
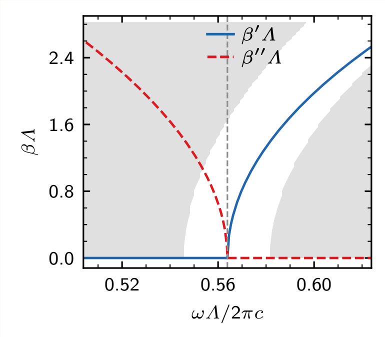


A photonic-crystal waveguide near cutoff



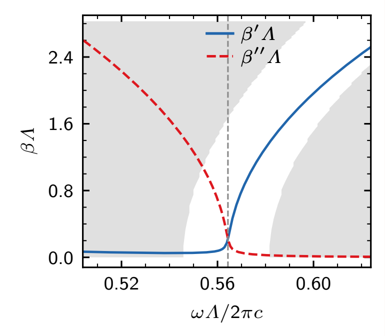
Analyte core between Bragg claddings (N periods $A|B$): the stop band replaces the PEC wall. The core mode has a **cutoff** ω_c ($\beta' = \beta'' = 0$) in the gap.

ideal, semi-infinite cladding



Square-root cutoff at ω_c.

finite, eight periods



Radiation loss modifies the cutoff.

Example: $\Lambda = 500\,$nm; $(d_A, d_B) = (250, 250)\,$nm; $(n_A, n_B) = (1.45, 2.10)$; air core $669\,$nm; TE even mode. Second-order gap: $\omega_c\Lambda/2\pi c = 0.564$, or $\omega_c = 2\pi \times 338.1\,$THz.

Loss sensitivity rises near cutoff, but is not equal to n_g

- Ideal, lossless: Γ_S diverges at cutoff; core loss limits the peak.
- Relative to PEC: reduced by the energy fraction in the analyte.
- Finite cladding: radiation also limits the peak.

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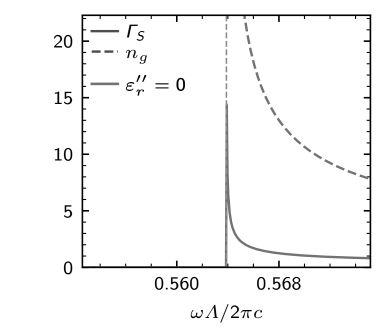

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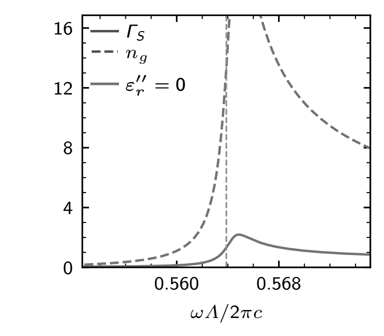

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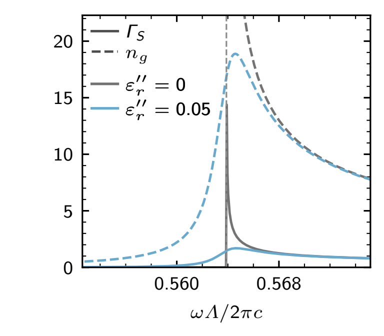

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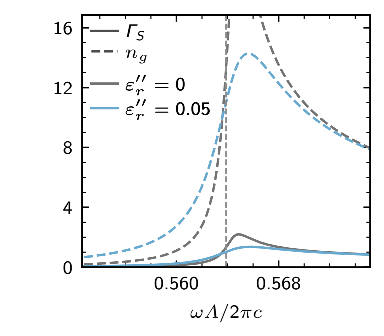

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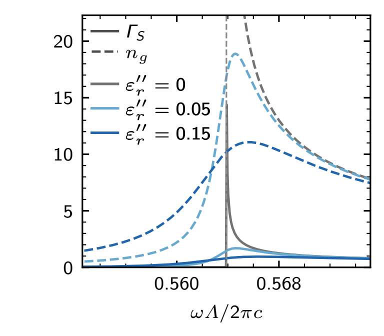

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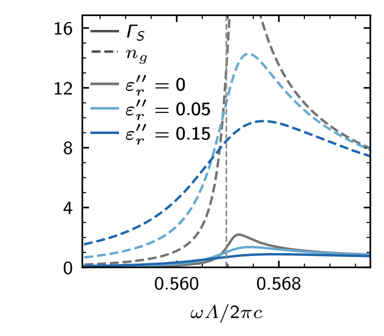


Filling fraction: lossless electric-energy fraction in the analyte core,

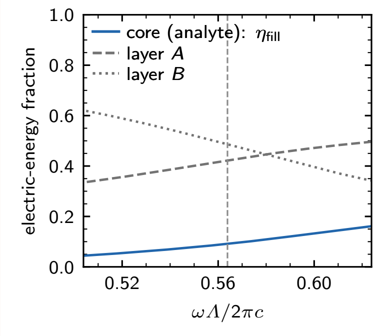
$$\eta_{\text{fill}} = \frac{\varepsilon_c \int_{\text{core}} |E|^2\, dx}{\int \varepsilon(x)\, |E|^2\, dx},$$

ε_c: core; $\varepsilon(x)$: transverse profile; x: transverse coordinate.

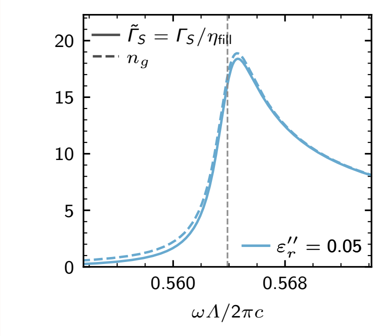
Normalised enhancement (plane-wave loss $\eta_{\text{fill}}\, \varepsilon_r''$):

$$\tilde{\Gamma}_S = \frac{\Gamma_S}{\eta_{\text{fill}}} = \frac{\mathcal{S}}{\eta_{\text{fill}}\, \mathcal{S}_{\text{ref}}}.$$

- Lossless closed guide: $\tilde{\Gamma}_S = n_g / n_{\text{core}}$.



ideal cladding



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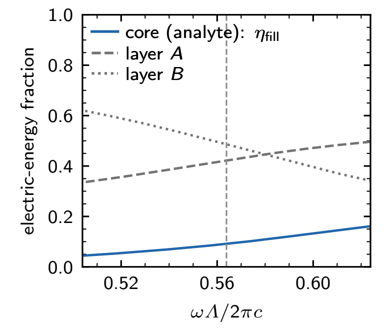

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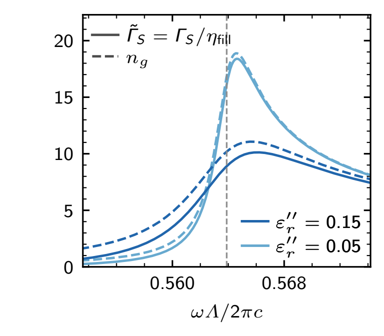


Material slow light: dispersive line and absorber

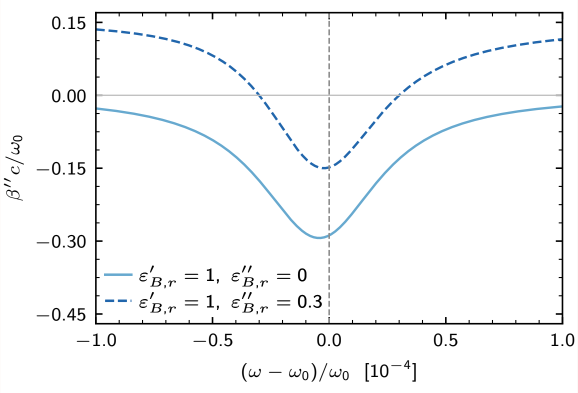
Plane wave ($k_\perp = 0$): background $\varepsilon_B = \varepsilon_0(\varepsilon'_{B,r} + \mathrm{j}\varepsilon''_{B,r})$, Lorentzian gain, and analyte:

$$\varepsilon(\omega) = \varepsilon_B + \varepsilon_0 \frac{G}{\omega - \omega_0 + \mathrm{j}\gamma_R} + \mathrm{j}\,\varepsilon_0\,\varepsilon''_{A,r}.$$

- background: non-dispersive, with loss $\varepsilon''_{B,r}$;
- gain (G, γ_R): pump-controlled slow light;
- analyte $\varepsilon''_{A,r}$: differentiation variable only.

$$n_g^{\mathrm{pw}} = n' + \omega \frac{dn'}{d\omega}, \qquad \frac{dn}{d\omega} = -\frac{(n^2 - n_B^2)^2}{2Gn},$$

$n_B^2 = \varepsilon_B/\varepsilon_0$, $\mathcal{S}^{\mathrm{pw}} = n'/(2|n|^2)$.
Example: $\gamma_R = 3 \times 10^{-5}\omega_0$ ($5.8\,$GHz half-width at $1.55\,\mu$m), $G/\gamma_R = 0.6$, set deliberately high [Y. Okawachi et al., Opt. Express 14, 2317–2322 (2006)].



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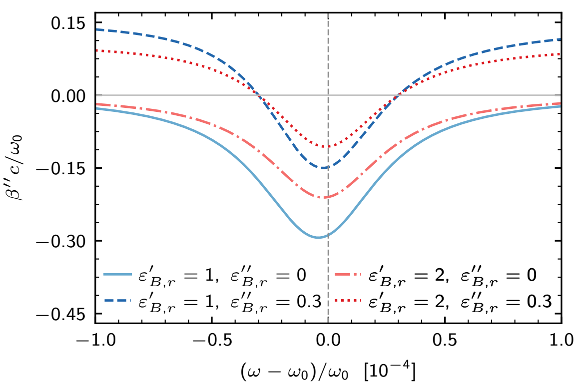
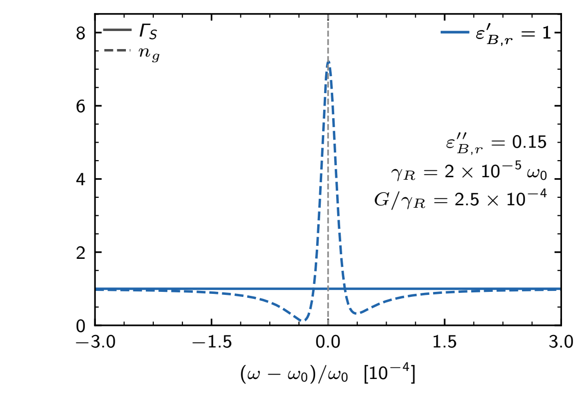


Large group index, unchanged loss sensitivity

- Narrow peak of n_g at the line centre.
- Both backgrounds: $|\Gamma_S - 1| < 10^{-4}$ over the plotted interval.



Experimental evidence

Brillouin slow light in a solid-core photonic-crystal fibre, acetylene in the holes: no enhancement observed.

[S. Chin et al., Adv. Opt. Sci. Congress (2009)]
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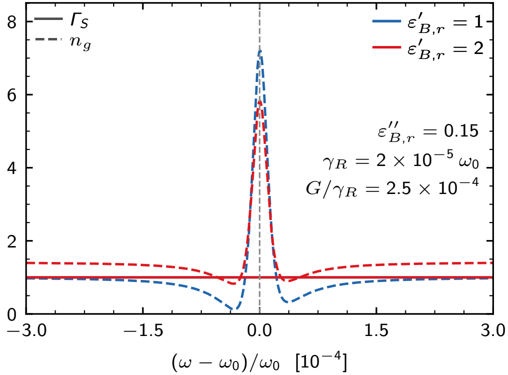

Why n_g is not a universal enhancement metric

Configuration	Origin of dispersion	What sets n_g	What sets $\mathcal{S}$	Result
PEC guide	waveguide cutoff	$1/\lvert n_{\text{eff}}\rvert$	$1/\lvert n_{\text{eff}}\rvert$	$\Gamma_S \simeq n_g/n'$ (low loss)
Photonic-crystal guide	waveguide cutoff (modified by leakage)	$1/\lvert n_{\text{eff}}\rvert$	$\eta_{\text{fill}}/\lvert n_{\text{eff}}\rvert$	$\Gamma_S \simeq \eta_{\text{fill}} n_g/n_{\text{core}}$ (low loss/leakage)
Lorentz gain line	$\varepsilon(\omega)$ resonance	$dn'/d\omega$	$\lvert n\rvert$	$\Gamma_S \simeq 1$ (line model)

Future work

1. **Energy balance with evanescence.** Extend $2\beta'' = \gamma_D / v_E$ to reactive stored energy through cutoff.
   [E. O. Schulz-DuBois, Proc. IEEE **57**, 1748–1757 (1969)]
   [G. Diener, Phys. Lett. A **235**, 118–124 (1997)]
2. **Open waveguides.** Separate absorption from radiation (for example in the ARROW case). Can $\partial\beta''/\partial\varepsilon_r''$ be negative?
3. **Other material resonances.** Test $|\Gamma_S - 1| \ll 1$ for EIT, Fano, and gain–loss media at $\mathcal{PT}$-symmetry breaking.
4. **Nonlinear enhancement.** Define a separate metric for processes governed by energy density and phase matching. [V. Eckhouse et al., Opt. Lett. **35**, 1440–1442 (2010)]

Conclusions

1. We introduce $\mathcal{S} = \partial\beta''/(k_0\partial\varepsilon_r'')$ and $\Gamma_S = \mathcal{S}/\mathcal{S}_{\text{ref}}$ to quantify Beer–Lambert enhancement.
2. Near a structural cutoff the mode is not rigorously but partly evanescent, and the same analyte loss coefficient produces more attenuation.
3. At low core loss and leakage, $\Gamma_S \simeq \eta_{\text{fill}} n_g / n_{\text{core}}$.

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Conclusions

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Thank you for your attention

Questions?

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References

[1] A. Beer, Ann. Phys. **162**, 78 (1852).

[2] J. Peatross, S. A. Glasgow, and M. Ware, Phys. Rev. Lett. **84**, 2370 (2000).

[3] S. Chin et al., in Adv. Opt. Sci. Congress (2009), SMA3.

[4] L. Thévenaz, I. Dicaire, and S. Chin, in Adv. Slow Fast Light V, Vol. 8273 (2012), p. 82731D.

[5] I. Dicaire et al., Opt. Lett. **37**, 4934 (2012).

[6] J. Grgić et al., Opt. Express **18**, 14270 (2010).

[7] H. G. Winful, Phys. Rep. **436**, 1 (2006).

[8] E. O. Schulz-DuBois, Proc. IEEE **57**, 1748 (1969).

[9] Y. Okawachi et al., Opt. Express **14**, 2317 (2006).

[10] G. Diener, Phys. Lett. A **235**, 118 (1997).

[11] V. Eckhouse et al., Opt. Lett. **35**, 1440 (2010).