Experimental Observation of Single- and Multi-Site Matter-Wave Solitons in an Optical Accordion Lattice

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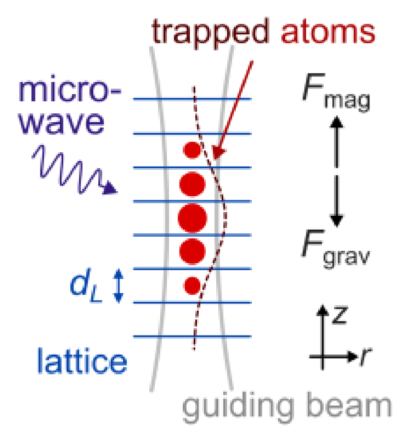
Summary: solitons of ultracold Cesium in a optical lattice

Platform. Cesium BEC in an **optical accordion lattice** with tunable spacing d_L and depth V_0.

Protocol. Prepare a wave packet over selected sites, quench $a_s < 0$, remove axial trap to form **single-site (SS)** and **multi-site (MS)** solitons.

- Cs BEC in a crossed dipole trap; interactions tuned via a broad Feshbach resonance with a zero crossing near 17.1 G.
- Transfer to a 780 nm accordion lattice and select 1–3 sites via microwave transfer and resonant light.
- Typical preparation: $N \sim 3 \times 10^4$ before selection, $N \lesssim 3 \times 10^3$ after selection.
- Quench a_s and release the axial trap in ~ 2 ms to trigger soliton formation.
- Image after evolution using a magnification ramp: increase V_0 and expand d_L to resolve site occupations.



Key results

- Stable SS window near $a_s \approx -8\,a_0$.
- MS solitons persist for 100–250 ms.
- Variational theory matches stability trends.
- 3D-GPE with loss captures collapse thresholds and reproduces atom-number dynamics.

Overview of the experimental conditions

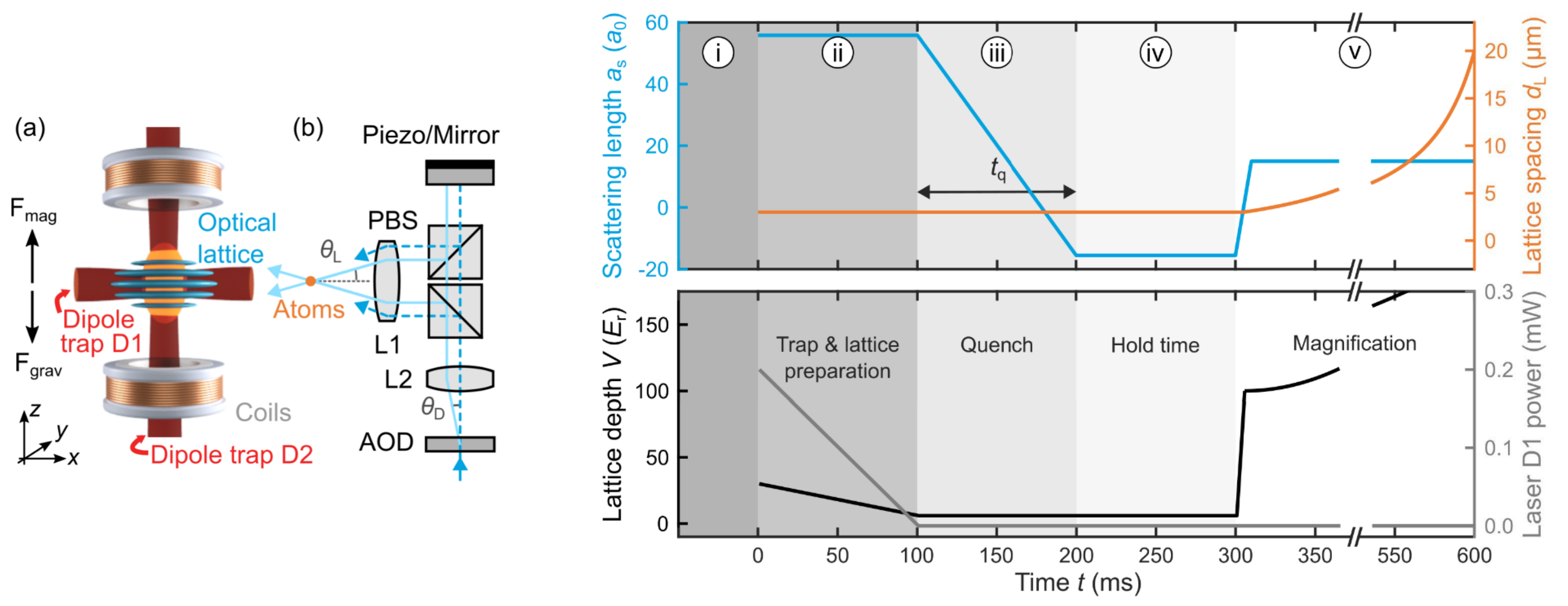


Figure 1. Trap and lattice geometry (left) and quench protocol (right). (From Ref. [2])

$$d_L = \frac{\lambda}{2\sin(\theta_L)}, \qquad E_r = \frac{(\hbar\pi/d_L)^2}{2m} = \frac{\hbar^2 k_L^2}{2m}.$$

- $d_L = 3.2(2)$ µm (prep)
- $V_0 \approx 100\,E_r$ (selection)
- $N \approx 1800$ (SS)
- $N \approx 2900$ (MS)
- $\omega_\perp \approx 2\pi \times (25–40)$ Hz
- t up to 250 ms (MS), 2 s (SS)

Site occupations N_j are extracted from absorption images after the magnification sequence.

- **SS measurement:** central-site fraction N_c/N_{tot} tracks localization and identifies stable windows.
- **MS measurement:** width w_m quantifies dispersion rates and stability over 100–250 ms.
- **Stability test:** compare measured dynamics with variational barriers E_{SS}, E_{MS} and 3D-GPE.
- **Loss dynamics:** early-time three-body loss reduces N, then slows as the packet expands.

Variational model and stability

We use a Gaussian variational ansatz to derive an effective energy function for SS/MS stability.

$$E = \int d\mathbf{r}\,\psi^*(\mathbf{r})\left[-\frac{1}{2}\nabla^2 + \frac{1}{2}(x^2+y^2) + U(z) + \pi g|\psi(\mathbf{r})|^2\right]\psi(\mathbf{r}), \qquad U(z) = -V_0\cos(2k_L z).$$

Gaussian ansatz is parametrized by η (axial lenght) and σ (transverse length):

$$\psi(\mathbf{r}) = \frac{1}{\pi^{3/4}\sigma\eta^{1/2}}\exp\left(-\frac{x^2+y^2}{2\sigma^2} - \frac{z^2}{2\eta^2}\right) \longrightarrow E(\eta,\sigma) = \frac{1}{2}\left(\frac{1}{2\eta^2} + \frac{1}{\sigma^2} + \sigma^2\right) + \frac{g}{2\sqrt{2\pi}}\frac{1}{\sigma^2\eta} - V_0\exp(-k_L^2\eta^2).$$

Minimizing over σ gives $E(\eta)$ with SS/MS minima and barriers B_{SS}, B_{MS}.

- Interaction parameter $g = 2a_s N/a_\perp$, lattice depth V_0 and lattice wave number $k_L = \pi/d_L$ enter the energy landscape. Predicts stability regions that guide experimental scans of a_s, V_0, and d_L.

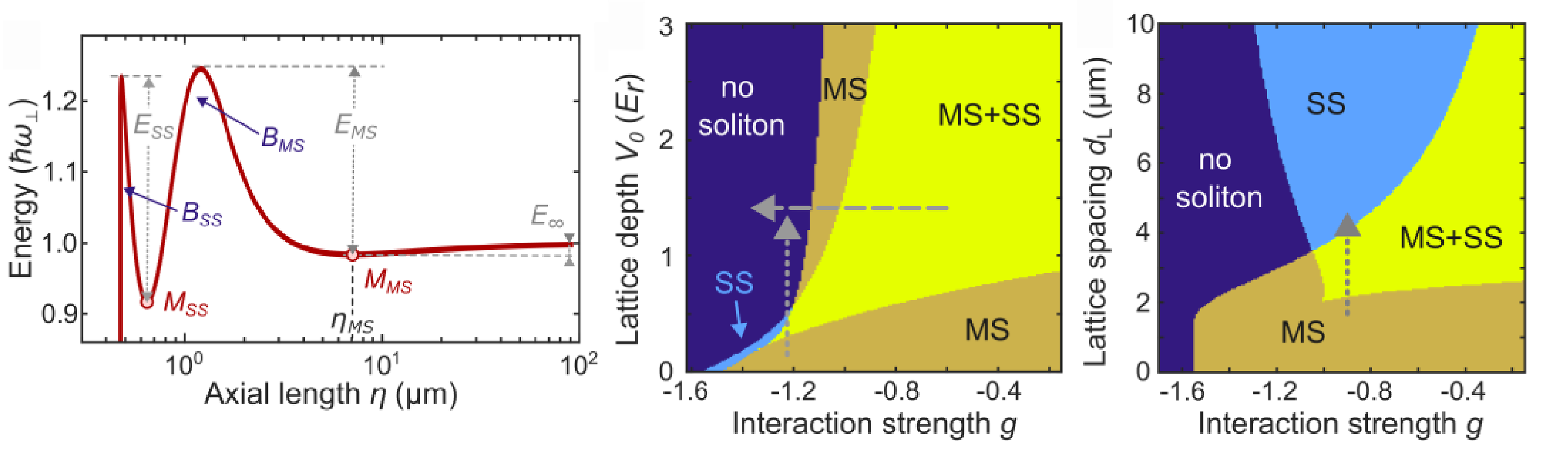


Figure 2. Variational stability map versus (g, V_0, d_L).

Single-site solitary waves

Three regimes appear: stable SS, dispersion near $a_s \approx 0$, and collapse for strong attraction. For $a_s \approx -8\,a_0$ the wave packet stays localized for up to ~ 2 s, while $a_s \lesssim -10\,a_0$ leads to collapse.

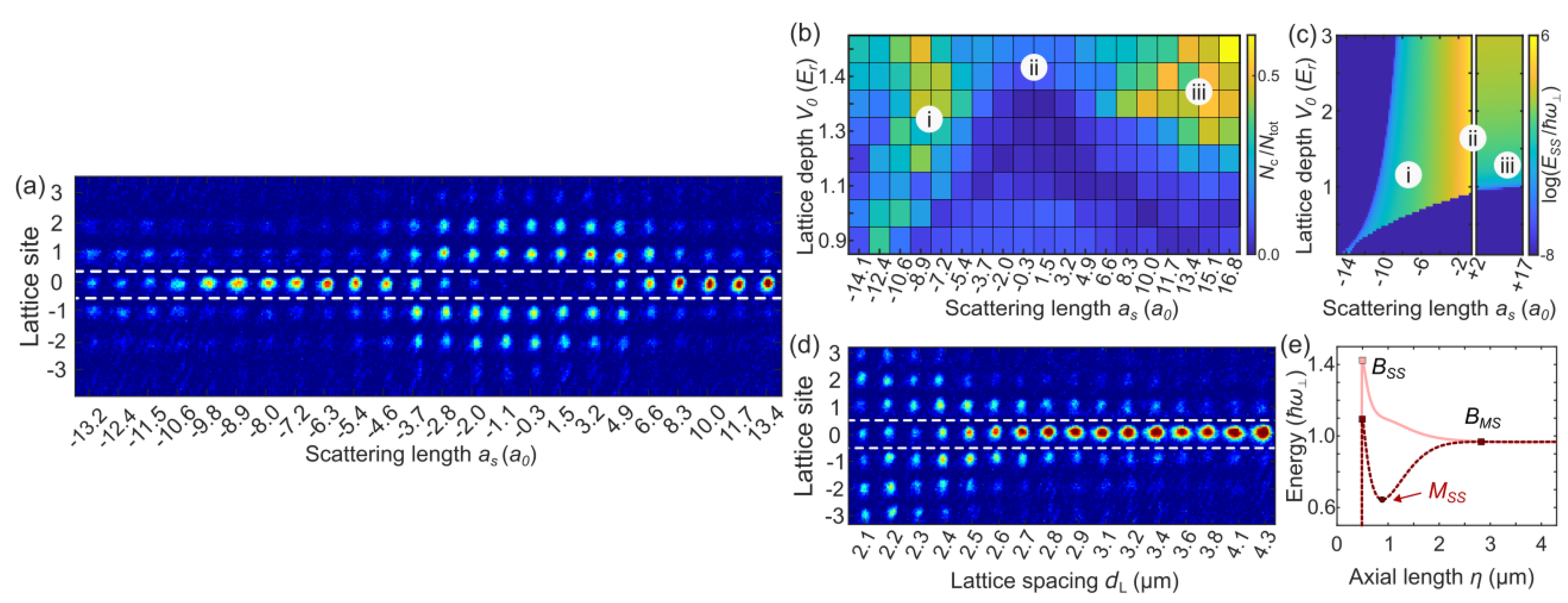


Figure 3. Stability of single-site solitons: density profiles and central-site fraction versus a_s, V_0, and d_L.

A stable SS window appears around $a_s \approx -8\,a_0$, while repulsive interactions produce self-trapping at large V_0.

Multi-site solitary waves

Multi-site packets are prepared from three adjacent sites and sets to a 3–5 site distribution. We track the width w_m and site occupations over 100–250 ms after the quench. For $a_s \approx +2.0\,a_0$ the packet disperses, while for $a_s \approx -5.7\,a_0$ it remains localized and soliton-like. Early-time atom loss is dominated by three-body recombination in high-density regions; subsequent loss slows as the packet expands.

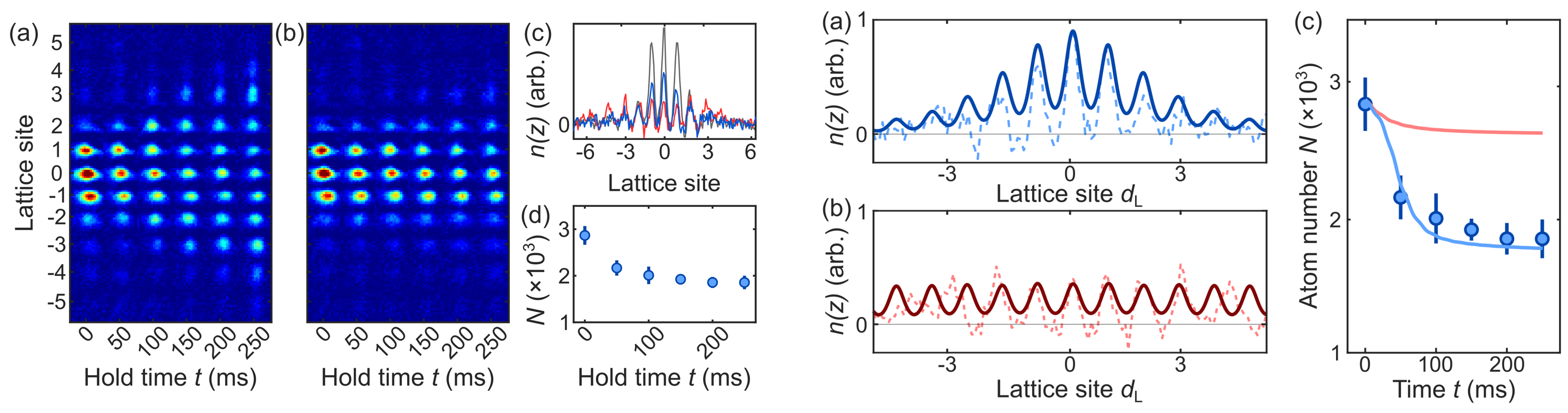


Figure 4. Time evolution and stability of MS solitons (left). Simulations with the 3D-GPE (right).

Comparison with Gross-Pitaevskii theory: collapse prevented by losses

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi + 2\pi\hbar^2 g|\psi|^2\psi - ig_5|\psi|^4\psi.$$

Mean-field dynamics with contact interactions $g = 2a_s N/a_\perp$ and an effective three-body loss term g_5. The density loss is parametrized by one-, two-, and three-body processes: three-body loss is dominant in the experimental conditions

$$\dot{n}(\mathbf{r},t) = -\frac{1}{\tau}n(\mathbf{r},t) - L_2 n(\mathbf{r},t)^2 - L_3 n(\mathbf{r},t)^3 \longrightarrow g_5 = \frac{\hbar N^2 L_3}{2}.$$

We use $L_3 = 5\times10^{-39}$–5×10^{-38} m^6 s^{-1} and $V(x,y,z) = \omega_z^2 mz^2/2 + \omega_\perp^2 m(x^2+y^2)/2 + V_0\cos(2k_L z)$. We compare data with 3D-GPE using these interaction and loss parameters; it captures the collapse/dispersion boundary and the measured dynamics of w_m and atom number. Calculations show two regimes: rapid density increase with strong loss for large negative a_s, and slow dispersion for weaker attraction.

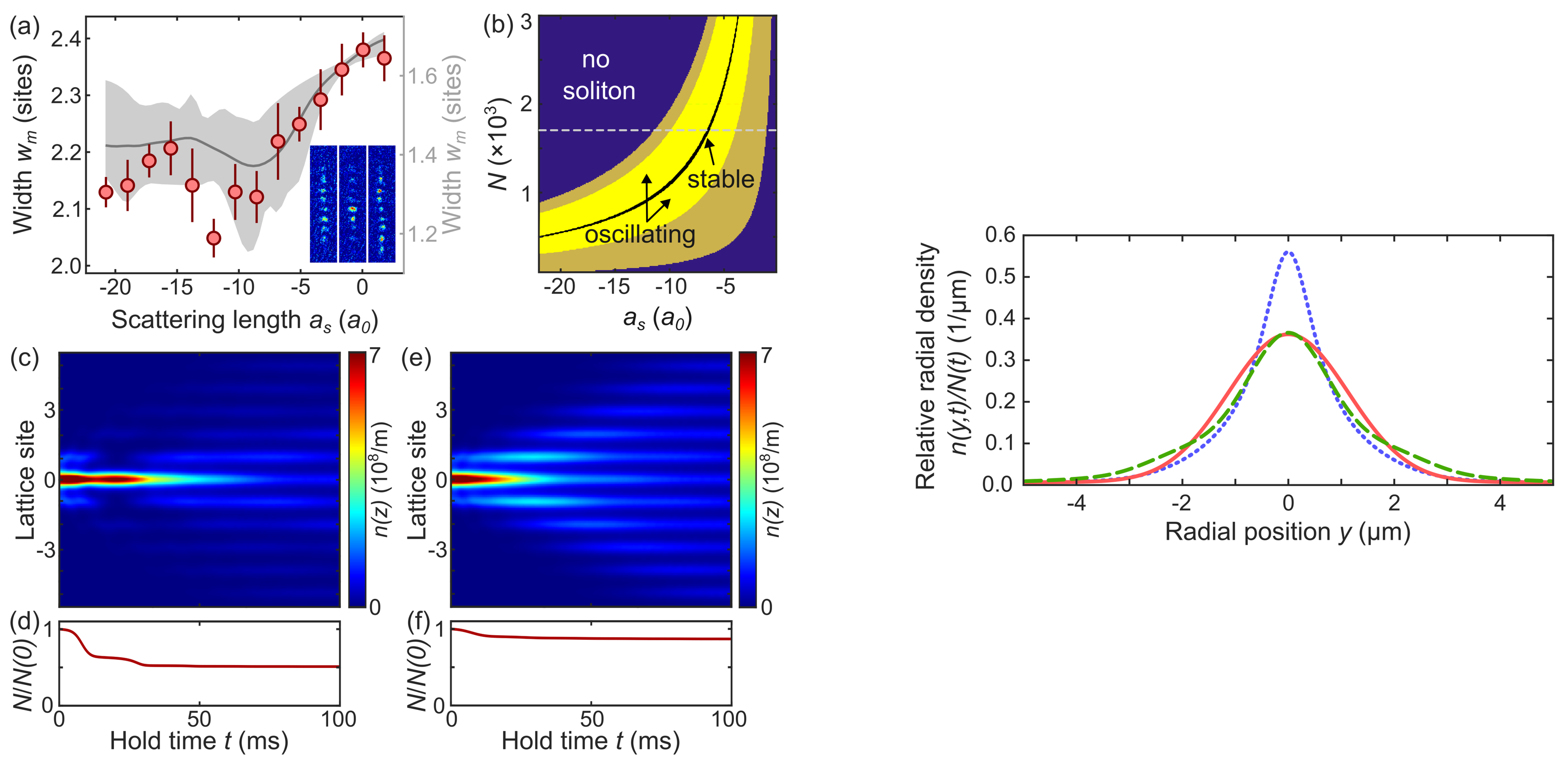


Figure 5. Collapse of multi-site wave packets: w_m vs a_s with 3D-GPE band (left) and transverse distribution from the numerical solution over the dynamics in the strongly attractive case (right).

Remarks and future directions

- Reduced models (NPSE and 1D GPE limits) offer a compact description of 3D dynamics; systematic benchmarks across (a_s, V_0, d_L) can map their validity.
- Full 3D-GPE simulations (code in Rust and Python is available: `https://github.com/lorenzifrancesco/rust-waves`) can already be used as a benchmark.
- The barrier picture provides rules for stability/decay; extending it with reduced models can guide fast parameter scans.
- Beyond SS/MS bright solitons, lattice gap solitons near band edges and repulsive-interaction self-trapped states may be investigated.

References

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