Pulse Collision Model and Physical Asymptotics for Fast Evaluation of Nonlinear Interference Noise in Few-Mode Fibers

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Why compute NLI using a time-domain model?

- Pulse collision model for FMF
- Computing collision integrals and noise variances
- Complexity challenges in FMF

Part 2: Model and results

Asymptotics and their inclusion into the general estimation method

- Asymptotic regimes (high/low DGD)
- General fitting model
- Case study: 4-mode group FMF

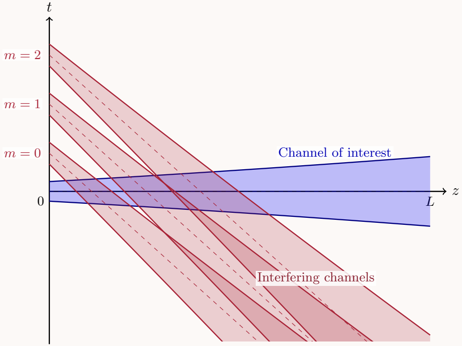
Part 1

Introduction

Why compute NLI using a time-domain model?

Pulse collision model for Few-Mode Fibers

- Time-domain perturbative approach for NLIN
- Channel of interest (COI) at $t = 0$, interfering channel (IC) walks off with DGD $\Delta\beta_1$
- Collision orders $m = 0, 1, 2, \ldots$ (initial temporal offsets mT)
- At low dispersion, the dominant NLIN contribution comes from pairwise **two-pulse** collisions
- Key variables:
  - L: fiber length, T: symbol period
  - β_{2a}, β_{2b}: chromatic dispersion of channels a and b
  - $\Delta\beta_1$: differential group delay (modal dispersion)
- In FMFs, $\Delta\beta_1$ is set by inter-modal dispersion, independent of β_{2a}, β_{2b}



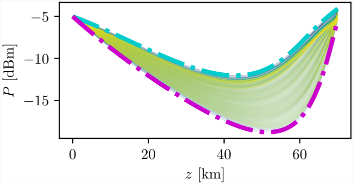
Computing the pulse collisions

Let a, b label mode groups and p, q WDM channel indices. XPM noise decomposes into same-group (SG, $a = b$) and cross-group (XG, $a \neq b$) contributions:

$$\sigma^2_{\mathrm{XPM}} = \sum_{q \neq p} \sigma^2_{\mathrm{SGXPM}} + \sum_{b \neq a} \sum_{q} \sigma^2_{\mathrm{XGXPM}}$$

We focus on **two-pulse** collisions, described by a spatial integral along the fiber:

$$X_{0mm} = \int_0^L dz\, f(z)\, I_m(z)$$



where the local collision integral measures pulse overlap:

$$I_m(z) = \int_{-\infty}^{\infty} dt\, |g^{(0)}_{ap}(z,t)|^2 \times |g^{(0)}_{bq}(z, t + mT - \Delta\beta_1\, z)|^2$$

Noise coefficient

For each channel pair, define:

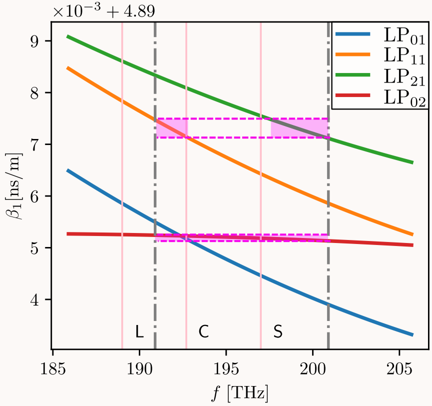
$$\mathcal{N}_{apbq} = \sum_m X^2_{0mm}$$

Both $\sigma^2_{\mathrm{SGXPM}}$ and $\sigma^2_{\mathrm{XGXPM}}$ scale *linearly* with $\mathcal{N}_{apbq}$.

- FMFs support multiple mode groups ($\mathrm{LP}_{01}, \mathrm{LP}_{11}, \mathrm{LP}_{02}, \mathrm{LP}_{21}$) – each with distinct group delay, as shown in the figure
- Large number of channel pairs: $\sim 6.4 \times 10^5$ (200 WDM $\times$ 4 mode groups)
- Inter-modal dispersion creates large DGD $\Rightarrow$ many collisions per pair:

$$M = L\Delta\beta_1/T \sim 10^3\text{–}10^4$$

- Direct numerical integration becomes prohibitively expensive
- Frequency-domain GN-model and EGN-model approaches account for a wider set of system components and have been extended to FMFs , but they lack clear asymptotic regimes to dynamically select which computations to perform



Group delay per unit length for the FMF [FL et al., *Journal of Lightwave Technology* 44, 4476–4490 (2026)]

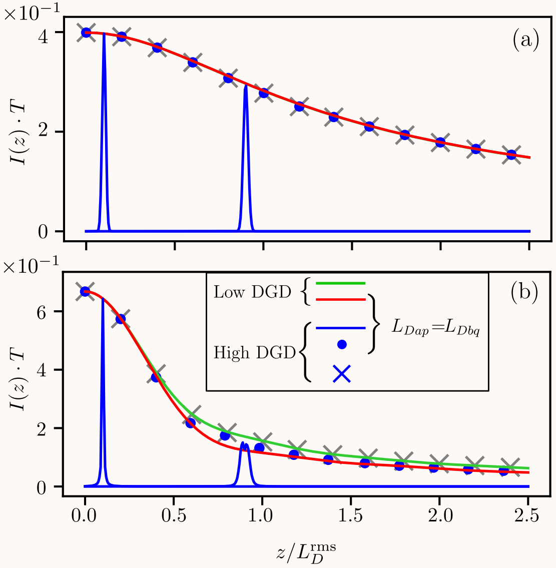
Part 2

Model and results

Asymptotics and their inclusion into the general estimation method

- $L_W = T/|\Delta\beta_1|$ (walk-off): high-DGD collisions are short and peak at $z_m = mL_W$
- $L_{Da} = T^2/|\beta_{2a}|$, $L_{Db} = T^2/|\beta_{2b}|$ (dispersion): peaks for identical (circles) or different (crosses) channel dispersion lengths
- $L_D^{\mathrm{rms}} = \sqrt{2}\, L_{Da} L_{Db} / \sqrt{L_{Da}^2 + L_{Db}^2}$: Gaussian depends only on this rms; Nyquist only for $z/L_D^{\mathrm{rms}} \lesssim 0.5$
- Low DGD (red/green): single collision along entire fiber
- Remarkably, same formulas give high-DGD peaks and low-DGD full shape

Computed with constant power profile ($f(z) = 1$, lossless fiber)



Key insight: analytical asymptotic limits exist for the collision integrals

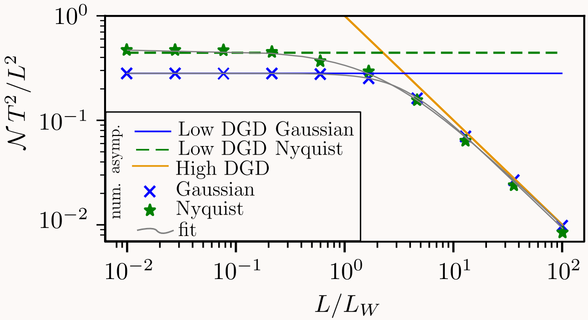
$$X_{0mm} \quad \longrightarrow \quad \begin{cases} \mathcal{N}^{\mathrm{HI}} \sim \dfrac{L}{T|\Delta\beta_1|} & \text{High DGD} \\ \mathcal{N}^{\mathrm{LO}} \sim \xi \left(\dfrac{L}{T}\right)^2 & \text{Low DGD} \end{cases}$$

High DGD regime ($L_W \ll L$)

- Many short collisions
- Dispersion-independent
- $X_{0mm} \approx 1/|\Delta\beta_1|$

Low DGD regime ($L_W \gg L$)

- Single collision along entire fiber
- Stronger NLIN
- Pulse-shape dependent (ξ)



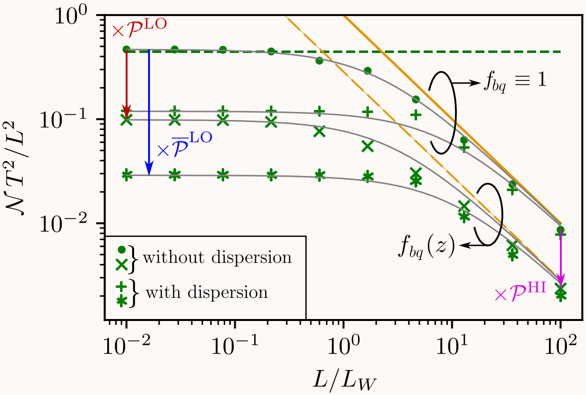
A universal fitting function interpolates between regimes:

$$\mathcal{N}\left(\frac{L}{L_W}\right) = \mathcal{N}^\circ \left(1 + \left(\frac{1}{\Lambda}\frac{L}{L_W}\right)^{1/\eta}\right)^{-\eta}$$

- $\mathcal{N}^\circ$: low-DGD asymptotic value
- Λ: transition DGD between regimes
- η: smoothness of transition
- High-DGD power law fixed to $\mathcal{N} \sim 1/|\Delta\beta_1|$

Realistic power profiles and dispersion are included via the correction factors below.

Corrections for dispersion and power evolution



Correction factors

Dispersion and power evolution (Raman amplification, losses) accounted via:

$$\mathcal{P}^{\mathrm{HI}} = \frac{1}{L}\int_0^L f(z)^2 dz$$

$$\mathcal{P}^{\mathrm{LO}} = \left(\frac{1}{L}\int_0^L f(z) dz\right)^2$$

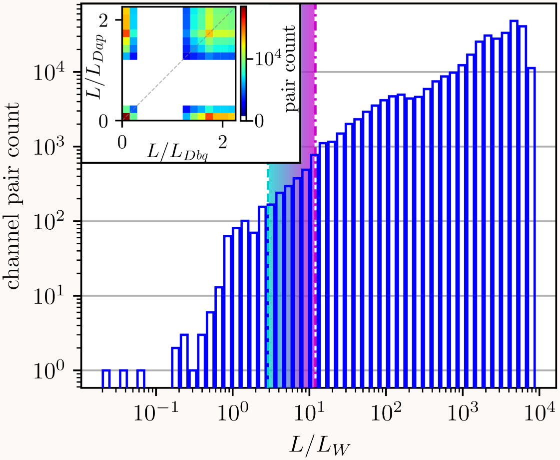
Applied to the ideal fitting parameters $(\mathcal{N}^\circ, \Lambda, \eta)$

FMF parameters

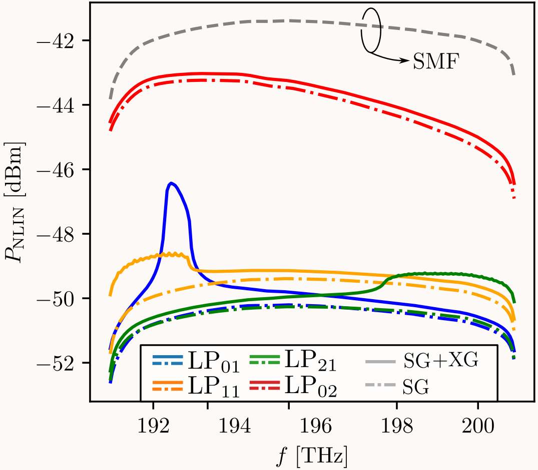
- $L = 70$ km, 33 Gbaud, 200 WDM channels per mode
- 6 counterpropagating Raman pumps
- Modes: $\mathrm{LP}_{01}, \mathrm{LP}_{11}, \mathrm{LP}_{21}, \mathrm{LP}_{02}$
- $A_{\text{eff}} = 220.7\ \mu\text{m}^2$, $n_2 = 2.7 \times 10^{-20}$ m^2/W
- Attenuation: 0.19 dB/km, spacing: 50 GHz, S+C+L band

How do the channel pairs distribute in DGD?

- Most channel pairs ($> 90\%$) are in high-DGD regime
- Only $\sim$ few % near GV matching (low DGD, higher NLIN)
- Dispersion values span $L/L_D \in [0, 2.3]$



NLIN: FMF vs SMF comparison



- FMF average NLIN is ~ 5.6 dB lower than SMF at same per-mode throughput
- However, NLIN varies widely across channels (~ 9 dB range) compared to SMF (~ 2.4 dB)
- GV-matched channel pairs suffer NLIN close to SMF levels
- Low-dispersion modes (LP_{02}) are the most affected

Computational efficiency

Steps: **S1**: ideal fitting model, **S2**: correction factors, **S3**: NLIN evaluation on all channel pairs

Evaluation cost of $I_m(z)$ (most intensive operation)

- **This work:** ~ 200 evaluations (S1) + ~ 80 evaluations (S2) + single integrals (S3)
- **Direct integration:** $\sim 6.4 \times 10^5$ channel pairs $\times 10^3$ collisions each

- S1 and S2 use multiprocessing
- S3 takes ~ 20 s on laptop hardware
- Direct integration would be prohibitively long

Software was not optimized for performance; further speed improvements are possible.

Conclusions

- Analytical asymptotic expressions for NLIN in low- and high-DGD regimes under ideal conditions
- Universal fitting model with corrections for Raman amplification & dispersion
- Accurate NLIN estimation for a 4-mode-group FMF
- FMF shows lower average NLIN than SMF, but with significant inter-channel variation

This presentation was based on [FL et al., *Journal of Lightwave Technology* **44**, 4476–4490 (2026)].

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Thank you for your attention!

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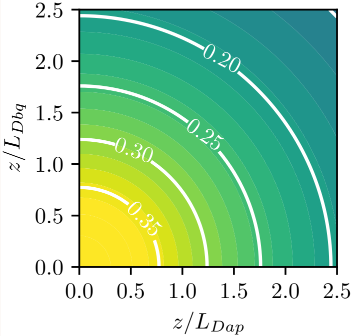
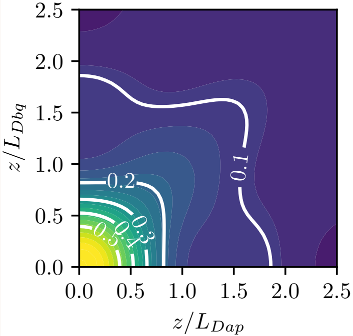
Part A

Additional material

Local collision integral for vanishing DGD



(a) Gaussian pulse



(b) Nyquist pulse

$I_{m=0}\,T$ as a function of z normalized to the single-channel dispersion lengths, for vanishing DGD

NLIN estimation pseudocode

Inputs : Channels of interest $\mathcal{A} = \{(a,p)\}$ for all mode groups a and WDM frequencies p

Link parameters: $L, T, \{\beta_2\}_\mathcal{A}, \{\Delta\beta_1\}_{\mathcal{A}\times\mathcal{A}}, \gamma, \kappa_{ab}, N_b$

$(L/L_D)_{\max} \equiv \max_{(a,p)} L\beta_{2,ap}/T^2$, heuristic m_T

Symbol statistics: $E_2 = \langle|b_0|^2\rangle$, $E_4 = \langle|b_0|^4\rangle$

Set of power profiles: $f_{bq}(z) = P_{bq}(z)/P_{bq}(0)$

Extremal power profiles: $f_{B,\min}(z) = \min_{bq} f_{bq}(z)$, $f_{B,\max}(z) = \max_{bq} f_{bq}(z)$

Pulse type

Output: List of $\{\sigma^2_{\mathrm{XPM}}\}_\mathcal{A}$

begin

S1 $(\mathcal{N}^\circ, \Lambda, \eta)_{\mathrm{ideal}} \leftarrow$ fitting Eq. (21) on pulse-specific dispersionless data with flat power profile (analytic or numeric)

S2 **foreach** $(a,p) \in \mathcal{A}$ **do**

- $\{\mathcal{P}^{\mathrm{LO}}\}_\mathcal{A} \leftarrow$ Eq. (26)
- $\{\mathcal{P}^{\mathrm{HI}}\}_\mathcal{A} \leftarrow$ Eq. (24)
- $\overline{\mathcal{P}}^{\mathrm{LO}}_{\min,\max} \leftarrow$ Eq. (27), with $I_m(z)$ computed up to $(L/L_D)_{\max}$ and $|m| \le m_T$

S3 **foreach** $(a,p) \in \mathcal{A}$ **do**

- $\mathcal{B} \leftarrow \mathcal{A} \setminus \{(a,p)\}$
- $\sigma^2_{\mathrm{XPM},ap} \leftarrow 0$
- **foreach** $(b,q) \in \mathcal{B}$ **do**
  - $\mathcal{P}^{\mathrm{HI}} \leftarrow$ from precomputed $\{\mathcal{P}^{\mathrm{HI}}\}_\mathcal{A}$
  - $\overline{\mathcal{P}}^{\mathrm{LO}} \leftarrow$ from precomputed $\{\mathcal{P}^{\mathrm{LO}}\}_\mathcal{A}$, $\overline{\mathcal{P}}^{\mathrm{LO}}_{\min,\max}$ and Eq. (28)
  - $(\mathcal{N}^\circ, \Lambda, \eta)_{\mathrm{corr}} \leftarrow (\mathcal{N}^\circ \overline{\mathcal{P}}^{\mathrm{LO}}, \Lambda \overline{\mathcal{P}}^{\mathrm{LO}}/\mathcal{P}^{\mathrm{HI}}, \eta)$
  - $\mathcal{N} \leftarrow$ Eq. (21) with $(\mathcal{N}^\circ, \Lambda, \eta)_{\mathrm{corr}}$ and $\Delta\beta_{1,apbq}$
  - **if** $b = a$ **then**
    - $C \leftarrow P^3T^2\gamma^2\kappa^2_{aa}\left[(E_4/E_2^2)(2N_a+3)-4\right]\mathcal{N}$ (Eq. (6))
  - **else**
    - $C \leftarrow 2N_bP^3T^2\gamma^2\kappa^2_{ab}\left[(E_4/E_2^2)-1\right]\mathcal{N}$ (Eq. (7))
  - $\sigma^2_{\mathrm{XPM},ap} \leftarrow \sigma^2_{\mathrm{XPM},ap} + C$

return $\{\sigma^2_{\mathrm{XPM}}\}_\mathcal{A}$

*From [FL et al., Journal of Lightwave Technology **44**, 4476–4490 (2026)]*