Propagation of Precursors in dispersive media

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Purpose of the presentation

1. Explain the need of the concept of precursor
2. Describe the most elementary, but significant, features of precursors theory
3. Comment two experiments found in literature
4. Explore the application possibilities of the phenomenon

Motivation

regions of strong dispersion

In a linear dispersive medium, the propagation of a uniform plane wave exibits a frequency-dependent wavenumber k.

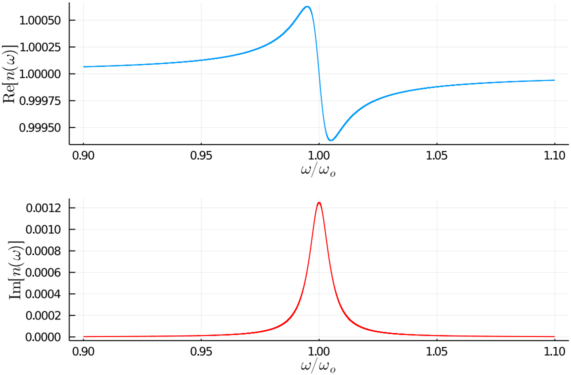
The dependance of k from ω (so called *dispersion relation*) can be approximated with a first order approximation in the case of propagation of a signal which is narrowband, by using the *group velocity* $v_g = d\omega/dk$.

In the case of resonances, the bounds on the signal bandwidth becomes much stricter, and the approximation is no more valid in general. Considering a Lorentz model for $\varepsilon_r(\omega)$ as an example, the group velocity is calculated.

Single resonance Lorentz medium

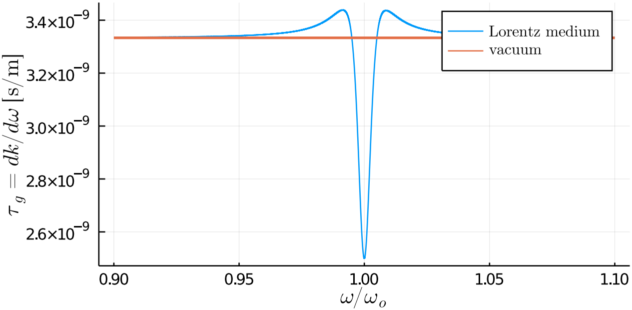
Recall Lorentz oscillator model for bound charges:

$$\varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 + j\gamma\omega} \qquad \text{and} \qquad n(\omega) = \sqrt{\varepsilon_r(\omega)}$$



Group velocity and signal velocity

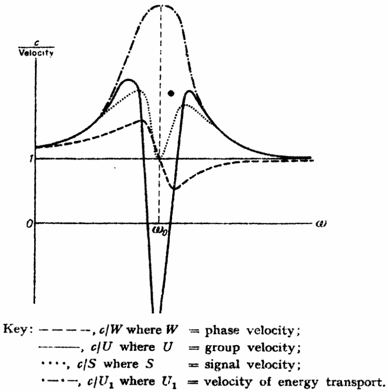
The group velocity can't be considered anymore as the speed of propagation of the signal, otherwise we would have a contradiction of *special relativity*. We are in need of a new definition of signal velocity.



Sommerfeld and Brillouin theory

In 1914, A.Sommerfeld addressed the problem of a *step modulated harmonic field* propagating at the resonance frequency.

The solution clarified the apparent inconsistency with relativity, and showed the propagation of waves, called precursors, with speed c. Later, the analysis was carried further by L.Brillouin.



All the original results are collected in Brillouin [4], plot from pg. 124

Mathematical comments

The propagation problem can be addressed in linear superposition terms:

$$H(L,\omega) = \exp\left[-j\frac{\omega}{c}\sqrt{\varepsilon_r(\omega)}L\right]$$

$$E(L,t) = \frac{1}{2\pi j}\int_\Gamma H(L,s)\ \mathcal{L}[E(0,t)](s)e^{st}ds$$

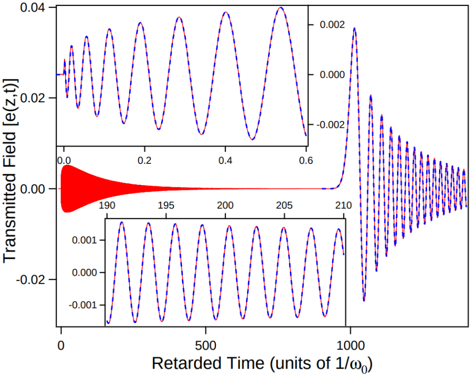
In general, no analytical solution is available.

By means of contour integration in the complex ω plane, two approximate (limit) solutions are found by Sommerfeld and Brillouin:

- *Sommerfeld precursor* for high frequencies $\omega >> \omega_0$
- *Brillouin precursor* for low frequencies $\omega << \omega_0$

Shape of precursors

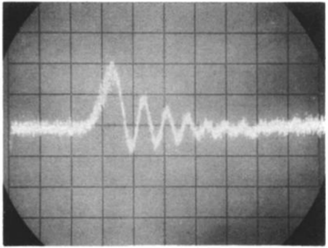
In a typical scenario with sufficiently long propagation distance, the two precursors are separated. The Sommerfeld precursor precede the Brillouin precursor.



plot from Macke, Ségard [1]

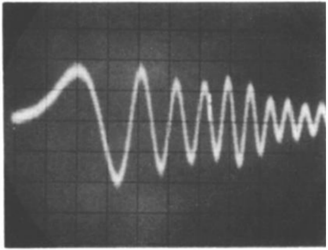
Microwave domain (1969 experiment)

It can be shown that the dispersion relation for certain waveguides can mimick the response of a Lorentz medium.



100 GAUSS

(5ns/div) 625MHz carrier over ferrimagnetic filled coaxial



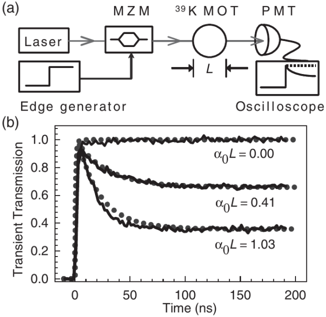
.2ns/div
5mv/div

100ps FWHM pulse over modified RG8/U line

plot from Pleshko, Palòcz [2]

Visible domain (2006 experiment)

In the optical domain the experimental techniques exploit the detection of the envelope of the signal. Even in this case the medium has a response very similar to the Lorentz model.



plot from Jeong, Dawes, Gauthier [3]

Applications and further topics

- As anticipated in [3], the propagation do not follow the usual Beer-Lambert attenuation law. This can be exploited for *imaging* applications, where deep penetration is required
- There are possible wireless communications applications [6]
- Propagation in *photonic crystals* is gaining interest [7]

Conclusions

- In the attempt to redefine signal velocity, Sommerfeld and Brillouin discovered the propagation of precursors around resonances
- Even if the mathematical framework is sophisticated, two approximate solutions are available for limit frequencies
- Due to lack of experimental apparatus, only recently some experiments confirmed the presence of electromagnetic precursors
- The literature suggest some interesting application behind the physical theory

References

1. Macke, Ségard. From Sommerfeld and Brillouin forerunners to optical precursors, *Phys. Rev. A* 87:043830 (2013).
2. Pleshko, Palòcz. Experimental Observation of Sommerfeld and Brillouin Precursors in the Microwave Domain, *Phys. Rev. Lett.* 22 (22) (1969).
3. Jeong, Dawes, Gauthier. Direct Observation of Optical Precursors in a Region of Anomalous Dispersion, *Phys. Rev. Lett.* 96(14):143901 (2006).
4. Brillouin. Wave propagation and Group velocity. *Academic press* (1960).
5. Aaviksoo, Kuhl, Plogg. Observation of optical precursors at pulse propagation in GaAs, *Phys. Rev. A* 44(9) (1991).
6. Alejos, Dawood, Medina. Experimental dynamical evolution of the Brillouin precursor for broadband wireless communication through vegetation. *Progress In Electromagnetics Research*, 111, 291–309, (2011).
7. Uitham, Hoenders. The electromagnetic Brillouin precursor in one-dimensional photonic crystals. *Optics Communications* 281(23), 5910-5918 (2008).

Approximate analytical solution

The following result is described in detail in [1]

Let

ω_c carrier frequency

L length of medium

α_0 maximum attenuation value (found at resonance)

$$t_B = \frac{2\alpha_0 \omega_p^2 L \gamma}{\omega_0^2}$$ delay between precursors

$$b = \left(\frac{\omega_0^2}{3 t_B} \right)^{1/3}$$

Approximate analytical solution

By means of special functions properities
Sommerfeld precursor:

$$E_S(L,t) \approx \frac{\omega_c}{\omega_0}\sqrt{\frac{t}{t_B}}\mathrm{J}_1(2\omega_0\sqrt{t_B t})e^{-2\gamma t}u(t)$$

where J_1 is the first Bessel function of the first kind Brillouin precursor:

$$E_B(L,t) \approx \frac{b}{\omega_c}\mathrm{Ai}(-bt')e^{-2\gamma t'}u(t')$$

where Ai is the Airy function, and $t' = t - t_B$.
$u(\cdot)$ is the Heaviside step.

Caveats

- Precursor phenomenon is not specific for EM waves (seismic, acoustic, etc.)
- The analytical solution is valid only with the step modulated sinusoidal wave input signal
- In a nonlinear media the behaviour of the precursor can be confused with other nonlinear phenomena (see commentary of water propagation experiment)

Negative velocity in anomalous dispersion